Lecture 10

In this lecture we continue our study of summation and product notation. Then we introduce matrix algebra.

Sol$^{\underline{n}}$

$$\sum_{i=0}^{4} 3^i = 3^0 + 3^1 + 3^2 + 3^3 + 3^4$$

$$= 1 + 3 + 9 + 27 + 81$$

$$= 121$$

Note

$$\sum_{i=0}^{n} ar^i = \frac{a(1-r^{n+1})}{1-r}$$

A geometric progression

$$= \frac{1(1-3^5)}{1-3}$$

$$= \frac{-243+1}{-2} = \frac{242}{2}$$

$$= 121$$

(Note: you don't have to memorise this formula for the sum of a geometric progression.)

Shift of summation index

$$\sum_{i=0}^{4} 3^i = \sum_{j=1}^{5} 3^{j-1}$$

Put $j = i+1$. $\therefore i = j-1$

$$\therefore \sum_{i=0}^{4} 3^i = \sum_{i=1}^{5} 3^{i-1}$$

$$= \sum_{i=1}^{5} \frac{1}{3} 3^i$$

$$= \frac{1}{3} \sum_{i=1}^{5} 3^i$$

$$\sum_{i=1}^{n} a f(i) = a \sum_{i=1}^{n} f(i)$$

(We can take a constant factor out the front of the summation sign.)

Product Notation

$$\prod_{i=1}^{n} a_i = a_1 \cdot a_2 \cdot \cdots \cdot a_n$$

$\prod$ stands for "product"

capital "Pi"

E.g. $$\prod_{i=1}^{7} i = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7$$

$$= 5040$$

$$= 7!$$

Exercises

① Evaluate $\sum_{i=0}^{4}(3i+1)$.

② Evaluate $\prod_{i=0}^{3} 2^i$.

③ Express $\sum_{i=0}^{4} 3^i$ as a summation starting at $i=2$.

Matrices

A *matrix* is an array of numbers or other objects, enclosed by round or square brackets.

Its *order* is $m \times n$, where there are m rows and n columns. The matrix is an $(m \times n)$-matrix.

E.g.

$$A = \begin{array}{cc} \begin{pmatrix} 1 & 0 & -3 \\ 0 & 2 & 5 \end{pmatrix} & \begin{array}{l} \text{— Row 1} \\ \text{— Row 2} \end{array} \\ \begin{array}{ccc} \text{Col. 1} & \text{Col. 2} & \text{Col. 3} \end{array} & \end{array}$$

This matrix has order 2×3.

A $(1 \times n)$-matrix is also called a <u>row vector</u>. An $(m \times 1)$-matrix is also called a <u>column vector</u>.

E.g. $\begin{pmatrix} 1 & 0 & -3 \end{pmatrix}$ — a row vector

$\begin{pmatrix} -3 \\ 5 \end{pmatrix}$ — a col. vector

Matrix algebra

We want to <u>add</u>, <u>subtract</u>, and <u>multiply</u> matrices.

Also, we want to <u>invert</u> matrices and to <u>transpose</u> them.

Two matrices can be added or subtracted if and only if they have the same order. The operations take place "entrywise".

Each no. or element in a certain row and a certain column is called an <u>entry</u> of the matrix.

For a matrix A, the entry in the i^{th} row and j^{th} column is often called a_{ij}. Then sometimes the whole matrix is written as

$$A = (a_{ij})_{1 \le i \le m,\, 1 \le j \le n}$$

$$= (a_{ij}) \quad \text{(in abbreviated form).}$$

Then if $A = (a_{ij})$ and $B = (b_{ij})$, where A and B have the same order, we get $A+B$ by adding corresponding entries a_{ij} and b_{ij}.

$$\begin{pmatrix} 1 & 0 & -2 \\ 0 & 3 & 5 \end{pmatrix} + \begin{pmatrix} 2 & 3 & 4 \\ -3 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 3 & 2 \\ -3 & 3 & 6 \end{pmatrix}$$

Subtraction works in a similar way.

What about mult$^{\underline{n}}$?

That's much more complicated!